Common Summation Identities in Statistics

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$$\sum_{i=1}^{n} a_i X_i \sum_{j=1}^{m} b_j Y_j = \sum_{i=1}^{n} \sum_{j=1}^{m} a_i b_j X_i Y_j$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\left(\sum_{i=1}^{n} X_i \right)^2 = \sum_{i=1}^{n} X_i^2 + \sum_{i=1}^{n} \sum_{j \neq i}^{n} X_i X_j$$

$$\left(\sum_{i=1}^{n} X_i \right)^2 = \sum_{i=1}^{n} X_i^2 + 2 \sum_{i=1}^{n} \sum_{i<j}^{n} X_i X_j$$

$$\sum_{i \neq j}^{n} a_i = \sum_{i=1}^{n} a_i - a_j$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=0}^{n} i^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$$

$$\sum_{i=m}^{n} i = \frac{(n+1-m)(n+m)}{2}$$

$$\sum_{i=m}^{n-1} a^i = \frac{a^m - a^n}{1-a}, (m < n)$$

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}, (|a| < 1)$$

$$\sum_{i=0}^{\infty} k a^i = \frac{a}{(1-a)^2}, (|a| < 1)$$