

Common Covariance Identities

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$$\begin{aligned}Cov(X, Y) &= E[X - E(X)][Y - E(Y)] \\Cov(X, Y) &= E[XY] - E[X]E[Y] \\Cov(aX, bY) &= abCov(X, Y) \\Cov(X, Y)^2 &\leq Var(X)Var(Y) \\Cov(X + a, Y) &= Cov(X, Y) \\Cov(a_1X_1 + a_2X_2, Y) &= a_1Cov(X_1, Y) + a_2Cov(X_2, Y) \\Cov(aX + b, cY + d) &= acCov(X, Y) \\Cov(X + Y, X - Y) &= Var(X) - Var(Y) \\Var(X + Y) &= Var(X) + Var(Y) + 2Cov(X, Y) \\Cov\left[\sum_{i=1}^n (a_iX_i + b_i), \sum_{j=1}^m (c_jY_j + d_j)\right] &= \sum_{i=1}^n \sum_{j=1}^m a_i c_j Cov(X_i, Y_j) \\ \sum_{i=1}^n \sum_{j=1}^n Cov(X_i, X_j) &= \sum_{i=1}^n Var(X_i) + \sum_{i=1}^n \sum_{j \neq i}^n Cov(X_i, X_j) \\Cov(X, Y) &= \sqrt{Var(X)}\sqrt{Var(Y)}Corr(X, Y) \\Cov(\sum_{i=1}^n X_i, \sum_{j=1}^n X_j) &= Var(\sum_{i=1}^n X_i) \\Var(\sum_{i=1}^n X_i) &= \sum_{i=1}^n Var(X_i) + 2\sum_{i < j}^n Cov(X_i, X_j)\end{aligned}$$